An Indicator of the Reliability of Analytical Structural Design

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A statistical analysis of structural static test failure data for major components of aircraft is presented. The data sample is divided into the static test failures of wings, fuselage, horizontal stabilizer, vertical stabilizer, and landing gear. The analysis results in the approximate determination of the specific statistical strength density function and cumulative distribution as a function of the rupture strength. Finally, the required factor of safety is computed for the "no static test" or analytical design case for the aforementioned components over a wide range of unreliabilities. It is concluded that components require factors of safety for the no static test or analytical design approximately an order of magnitude above the usual standard and would be prohibitive if implemented.

Nomenclature

a = statistical parameter in truncated Weibull distribution
b = statistical parameter in truncated Weibull distribution
F = function
m = inverse of the factor of safety
P = probability distribution function
R = reliability
S = strength
x = strength or load variable
γ = ($= \sigma/\mu$) scatter coefficient or coefficient of variation
σ = standard deviation
ϕ = normal probability density function
Φ = normal cumulative probability function
μ = mean

Introduction

In the formulation of structural design criteria, there is a constant need for knowledge of the reliability of stress analysis. One measure or indicator of this reliability can be gained by analyzing the statistical parameters for static test failures of major components of aircraft.

An analysis of static test failure data for aircraft was published by Jablecki.[1] This document, however, did not include a formal statistical analysis of data contained therein. The purpose of this paper is to present a simple statistical analysis of the Jablecki data and to indicate that reliance on pure analytical capability without testing would require design factors which are prohibitive.

The main objective of Jablecki's dissertation was to indicate the failure expectancy of the average airplane and identify the most frequent cause of failure during static testing of major aircraft components. A second objective was to suggest further investigation to uncover prevalent design errors.

In the Jablecki study, structural reliability based upon design analysis is compared with structural reliability based on both design analysis and static tests. No attempt was made by Jablecki to identify the reasons underlying each failure (i.e., design error, material error, human error, theoretical error, etc.) and such an identification is beyond the scope of this paper. In addition, his analysis indicates a measure of the reliability of stress analysis and associated skills in all but the most simple designs. The Jablecki data have special application to structural reliability problems. A statistical analysis of the data can be used to formulate a rational factor of safety for the case in which strength analysis is not verified by structural test.

The Jablecki dissertation was the first complete and comprehensive documentation of premature structural failures in static test situations. It is the hypothesis of this paper that these data are generally applicable to present day aircraft. This hypothesis is reasonable even when the better theoretical stress analysis methods developed in the last decade are taken into account; these improved methods are counterbalanced by the unknown hyperenvironments resulting from higher aircraft speeds and altitudes. Therefore, since no similar comprehensive data analysis has been accomplished for modern high performance aircraft to date, the Jablecki data will be utilized to assist in formulating structural design criteria.

The relative unreliability of structural analysis disclosed by the Jablecki data has made necessary a departure from the traditional concept of structural design criteria. Bouton and others[2-5] have studied the implications of analytical errors in the framework of structural reliability goals and have developed procedures for dealing with this problem in the formulation of structural design criteria. The present paper represents an element in the process of evolving new concepts of structural design criteria based on structural reliability and systems concepts.

Analysis

Reference 1 summarizes the results of static ultimate strength tests made by the United States Air Force on aircraft at Wright-Patterson Air Force Base during the time period 1940–1949. The study summary categorizes failures by major components of the aircraft. The principal components are wing, fuselage, horizontal stabilizer, vertical stabilizer, and landing gear (It should be pointed out that minor components and subassemblies could have been tested prior to their assembly into major components; however, no mention of this possibility is made in Ref. 1.) The failures are classified according to frequency of failure and the percentage of design ultimate load (L) at which failure occurred. L is defined as the product of a factor of safety and the limit load which the vehicle can be expected to encounter in actual service.

The problem in analyzing the Jablecki data is that for a relatively small sample, the probability distribution and its parameters must be derived. It can be shown that there are an infinite number of possible distributions which potentially fit any arbitrary set of points which are not a total

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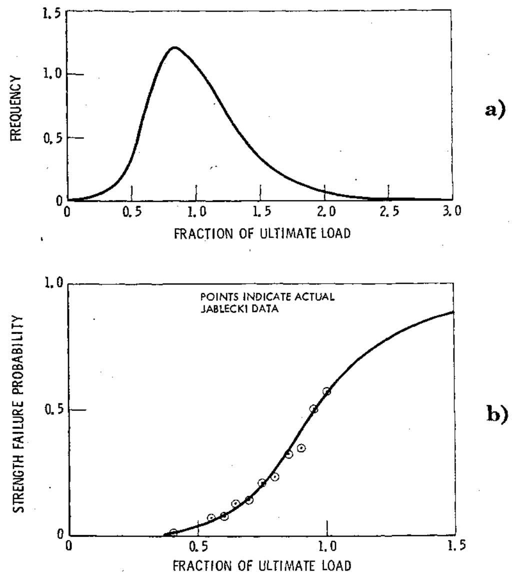

Fig. 1 Wing distributions: a) wing failure probability density distribution; b) wing cumulative probability distribution.

population. However, the distributon (or distributions) sought are in the class of all distributions having zero probability at or near zero load and a failure probability of one as the load becomes large (i.e., approaches a load somewhat above the load equivalent to the allowable strength). This fact materially reduces the number of potential distributions which could yield a compact (best fit) representation of the data.

Examination of the Jablecki data (Figs. 1–5) seems to show a natural division between the larger and smaller major components. Analysis indicates that the wing and fuselage (here called Jablecki Class I) data fit the log-normal distribution. The horizontal stabilizer, vertical stabilizer and landing gear data (called Jablecki Class II) best fit a truncated asymptotic Weibull distribution. These distributions were found to fit the data best based on chi-square tests of the several distributions examined. The log-normal and truncated asymptotic Weibull distributions were also very convenient in that the density or cumulative distribution functions and their parameters can be generated analytically, thus requiring a minimum of effort.

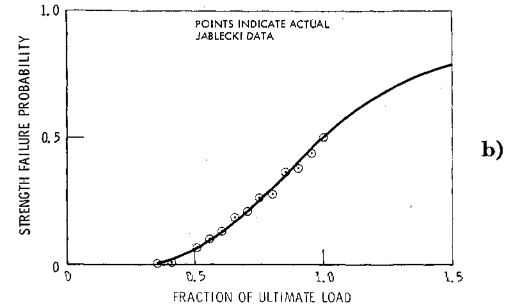

Fig. 2 Fuselage distributions: a) fuselage failure probability density distribution; b) fuselage cumulative probability distribution.

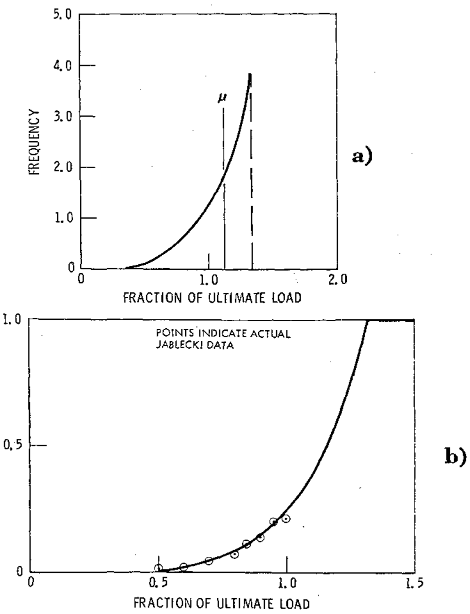

Fig. 3 Horizontal stabilizer distributions: a) horizontal stabilizer failure probability density distribution; b) horizontal stabilizer cumulative probability distribution.

Jablecki Class I Distribution

The method of fitting the log-normal distribution[6] to the wing and fuselage data is as follows.

Given any two points from the Jablecki data (or any curve of this general form), these points may be designated (x_1, P_1) and (x_2, P_2) where x_i is the value of the percentage of ultimate load at the ith probability and P_i is the value of the ith probability. Now the ith probability is given by

$$P_i = P[X \leq x_i] = \Phi\left(\frac{\ln x_i - \ln \mu_x/(1 + \gamma_x^2)^{1/2}}{[\ln(1 + \gamma_x^2)]^{1/2}}\right) \quad (1)$$

where μ_x is the value of the mean load, γ_x is the scatter coefficient (or coefficient of variation), defined as the ratio of the standard deviation (σ_x) to the mean load (μ_x), and Φ is the standard normal distribution function. The inverse of Eq. (1) yields,

$$\frac{\ln x_i - \ln \mu_x/(1 + \gamma_x^2)^{1/2}}{[\ln(1 + \gamma_x^2)]^{1/2}} = \Phi^{-1}(P_i) \quad (2)$$

Then for the given points (x_1, P_1) and (x_2, P_2),

$$\ln x_1 - \ln \mu_x + \tfrac{1}{2}\ln(1 + \gamma_x^2) = [\ln(1 + \gamma_x^2)]^{1/2}\Phi^{-1}(P_1) \quad (3a)$$

and

$$\ln x_2 - \ln \mu_x + \tfrac{1}{2}\ln(1 + \gamma_x^2) = [\ln(1 + \gamma_x^2)]^{1/2}\Phi^{-1}(P_2) \quad (3b)$$

then solving Eqs. (3a) and (3b) for γ_x yields,

$$\gamma_x = \{-1 + \exp[\ln x_2/x_1/\Phi^{-1}(P_2) - \Phi^{-1}(P_1)]^2\}^{1/2} \quad (4)$$

From Eqs. (3b) and (4), we can find the mean μ_x

$$\mu_x = x_2(1 + \gamma_x^2)^{1/2}\exp\{-[\Phi^{-1}(P_2)[\ln(1 + \gamma_x^2)]^{1/2}]\} \quad (5)$$

then the cumulative probability distribution can be found from

$$F(x) = \int_{-\infty}^{\frac{\ln(x/\mu_x)(1 + \gamma_x^2)^{1/2}}{[\ln(1 + \gamma_x^2)]^{1/2}}} \phi(t)dt \quad (6)$$

Table 1 Distribution parameters of the Jablecki data

Component	μ_x	γ_x
Wings	1.046	0.375
Fuselage	1.110	0.485
Horizontal stabilizer	1.150	0.139
Vertical stabilizer	1.280	0.189
Landing gear	1.110	0.167

where

$$\phi(t) = [1/(2\pi)^{1/2}] \exp - t^2/2$$

and

$$F'(x) = \frac{1}{x[\ln(1 + \gamma_x^2)]^{1/2}} \phi \left(\frac{\ln(x/\mu_x)(1 + \gamma_x^2)^{1/2}}{[\ln(1 + \gamma_x^2)]^{1/2}} \right) \quad (7)$$

On this basis, we can find the parameters and hence additional points on the best fit probability density and cumulative probability. The values of γ_x and μ_x were determined using many pairs of values (x_i, P_i) and in each case the values were found to agree closely.

Jablecki Class II Distribution

The truncated asymptotic Weibull[7] fitting method is derived by again choosing any two points (x_1, P_1) and (x_2, P_2) as defined earlier on the Jablecki cumulative distribution curve (or any other general curve desired).

The distribution will be derived under the assumptions specified below,

$$\begin{aligned} F(x; a, b) &= (x/a)^{1/b}, 0 \leq x \leq a \\ &= 0, x < 0 \\ &= 1, x > a \end{aligned} \quad (8)$$

Now, for the ith probability, let

$$P_i = F(x_i; a, b) = (x_i/a)^{1/b}, 0 < P_i < 1 \quad (9)$$

Replacing x_i, P_i by their values at points 1 and 2, taking logarithms, and solving for ln a and b yields,

$$\ln a = \ln P_2 \ln x_1 - \ln P_1 x_2 / \ln P_2 - \ln P_1 \quad (10)$$

$$b = \ln x_1 - \ln a / \ln P_1 \quad (11)$$

The values of a and b were determined using many pairs of values (x_i, P_i) and in each case the computed parameters were found to agree within a small tolerance.

Results

Using the analysis presented previously, we can find the parameters of the five Jablecki curves.[1] These parameters are presented in Table 1.

The values of these parameters, computed from the original Jablecki data, are given in Figs. 1–5. Figures 1 and 2 give the cumulative probability fit using the log-normal distribution scheme (wing and fuselage). In Figs. 3–5, the truncated asymptotic Weibull distribution scheme is used to fit the cumulative probability data for empennage and landing gear structures. The probability density functions for each curve have been derived and are also presented (Figs. 1–5). The data base shown in the figures represents static tests on a total of 115 individual airplanes over the time period from 1940–1949. Many of the points shown on each curve are representative of a multiplicity of tests. In Fig. 1, the total sample tested was 99 wings. Ninety-four tests were the fuselage sample size. The sample size of horizontal stabilizers, vertical stabilizers, and landing gear are 91, 88, and 78, respectively.

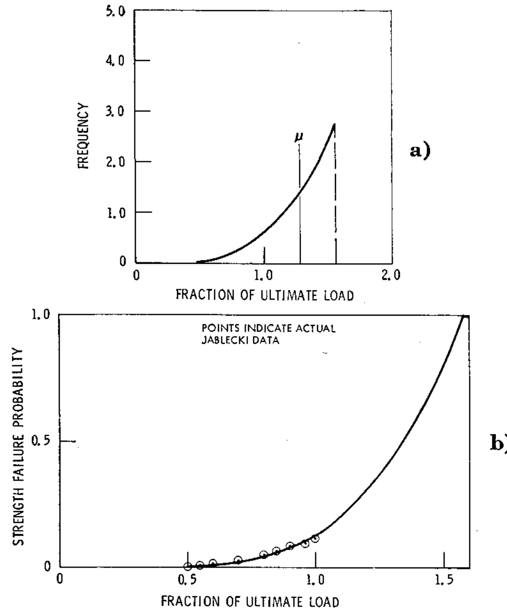

Fig. 4 Vertical stabilizer distributions: a) vertical stabilizer failure probability density distribution; b) vertical stabilizer cumulative probability distribution.

The curve fits given in Figs. 1–5 have several important uses in structural design criteria development. These figures give the expectation that a given structure will successfully sustain ultimate load on the first test. More important, they describe the reliability that a structure possesses prior to the static test. This type of information is extremely useful in structural reliability analyses. A further example of the utility of the Jablecki data is that approximate factors of safety can be derived which would have guaranteed a given level of structural reliability for the structures tested. In Ref. 2, the factor of safety is defined as the number by which a specified limit load is multiplied to compute the ultimate load a structure must sustain, without catastrophic failure, to provide a specified level of reliability

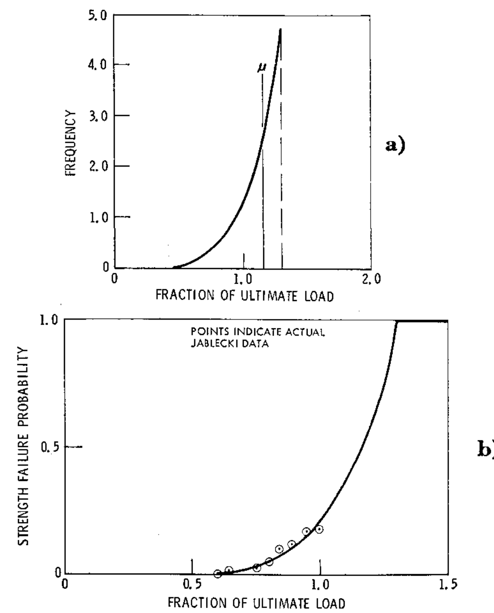

Fig. 5 Landing gear distributions: a) landing gear failure probability density distribution; b) landing gear cumulative probability distribution.

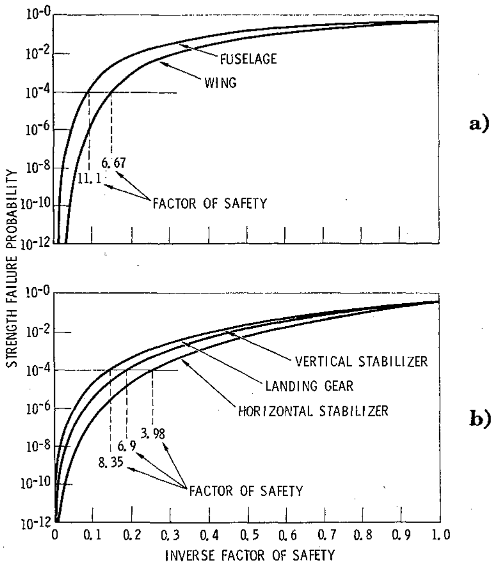

Fig. 6 Strength failure probability as a function of inverse factor of safety: a) Jablecki class I; b) Jablecki class II.

at the specified limit load. It should be pointed out that the curve fits in Figs. 1–5 have been extrapolated significantly beyond the actual data and care must be taken in using these curves beyond fraction of ultimate load values greater than one (1.0). Log-normal distributions (wing and fuselage) for unreliability vs the inverse of the factor of safety are presented in Fig. 6a. The inverse is used for convenience in displaying the wide range over which the factor of safety varies for the unreliabilities of interest.

The factor of safety is defined initially in terms of two strength levels, the design allowable strength S_1 and the test, or true strength S_2. Success (no failure during the test) is defined as that condition where

$$S_2 \geq mS_1 \tag{12}$$

and m is the inverse factor of safety. The reliability of a given test specimen will then be given by

$$R(m) = P(S_2 \geq mS_1) \tag{13}$$

From Eq. (12) we recognize that for any test specimen $S_2 \geq mS_1$ is required for success, or $S_2 < mS_1$ is expected for failure. These are the only possibilities, since from probability theory we have

$$P(S_2 \geq mS_1) + P(S_2 < mS_1) = 1 \tag{14}$$

Then, using Eq. (13), Eq. (14) can be restated as

$$1 - R(m) = P(S_2 < mS_1) \tag{15}$$

where $1 - R(m)$ is called the unreliability of the structure. For the log-normal distribution, then

$$\ln S_2 < \ln mS_1 \tag{16}$$

or

$$\ln S_2 - \ln S_1 < \ln m \tag{17}$$

so Eq. (15) becomes

$$1 - R(m) = P(S_2 < mS_1) = P(\ln S_2 - \ln S_1 < \ln m) \tag{18}$$

where $\ln S_1$ and $\ln S_2$ are independently and normally distributed (for the log-normal case) with the same mean and variance $\ln(1 + \gamma_x^2)$. Then the variance of $\ln S_2 - \ln S_1$ (also normally distributed) is given by

$$\sigma^2_{\ln S_2 - \ln S_1} = \sigma^2_{\ln S_2} + \sigma^2_{\ln S_1} = 2\ln(1 + \gamma_x^2) \tag{19}$$

where the following relationship has been utilized,

$$\sigma^2_{x-y} = \sigma_x^2 + \sigma_y^2$$

Then Eq. (18) is given by

$$P\left\{\frac{\ln S_2 - \ln S_1}{[2\ln(1+\gamma_x^2)]^{1/2}} < z\right\} = \int_{-\infty}^{Z} \phi(t)dt = \Phi(z) \tag{20}$$

And using Eq. (17) in Eq. (18),

$$1 - R(m) = P\left\{\frac{\ln S_2 - \ln S_1}{[2\ln(1+\gamma_x^2)]^{1/2}} < \frac{\ln m}{[2\ln(1+\gamma_x^2)]^{1/2}}\right\} \tag{21}$$

Using this result in Eqs. (18) and (20), Eq. (21) becomes

$$1 - R(m) = \Phi\{\ln m/[2\ln(1+\gamma_x^2)]^{1/2}\} \tag{22}$$

Solving Eq. (22) for the inverse of the factor of safety, m, for the log-normal distribution we obtain,

$$m = \exp\{[2\ln(1+\gamma^2)]^{1/2}\Phi^{-1}(1 - R)\} \tag{23}$$

where $\Phi^{-1}(1 - R)$ is defined in Eq. (2) and is a negative number when $R > 0.5$.

For the truncated asymptotic Weibull distribution, we derive the inverse factor of safety in the same manner as for the log-normal distribution. Equation (15) is restated,

$$1 - R(m) = P\{X < mx\} \tag{15}$$

we then have

$$P\{X < mx\} = \frac{1}{2}\int_0^a d\left(\frac{mx}{a}\right)^{1/2} = \frac{m^{1/b}}{2} \tag{24}$$

and using Eq. (15), the inverse factor of safety is obtained:

$$m = [2(1 - R)]^b \tag{25}$$

where

$$R > 0.5 \text{ and } b = \gamma^2 + (\gamma^4 + \gamma^2)^{1/2}$$

Plotted in Fig. 6b are curves of the inverse factor of safety as a function of unreliability using Eq. (25) for the horizontal and vertical stabilizers and the landing gear. This permits selection of a given factor of safety for a given component at a specified unreliability.

Conclusions and Recommendations

Jablecki, in his dissertation, concludes by indicating that 1) the stress analyst will improve the accuracy of his work by a more judicious use of conservative design factors; 2) further study of the structures which had failed and of the pertinent stress analyses would undoubtedly disclose much additional information on the source of errors which would assist the stress analyst in solving his many and varied problems; 3) it is highly recommended that the stress analyst become aware of typical failures so as to minimize their frequency; and 4) static test data contain a wealth of information which should be investigated in depth to improve the methods of stress analysis.

Examination of the wing and fuselage curves indicates that for these components we would need, without tests, factors of safety of 6.67 and 11.1, respectively to insure a typical conditional unreliability of 10^{-4} at limit load. This unreliability value corresponds to a reliability of 0.9999 which can be termed representative of the level required of aircraft. The use of a reliability factor of 0.9999 is motivated by the fact that it lies between the 0.99 factor for high-risk aircraft and the 0.999999 factor for low-risk aircraft which are the usual standards. It is apparent then, that the limit factor of safety required to give the desired reliability would be impossibly high compared to normal usage unless static tests were run to disclose and eliminate errors. Using the same unreliability of 10^{-4} for the Jablecki Class II structures as before, it is immediately seen that without tests the factors of safety for the horizontal stabilizer, vertical stabilizer, and the landing gear are 3.98, 6.9, and 8.35, respectively. It can then be concluded that Jablecki's analysis indicates that static testing is a major factor in the total structural system analysis and qualification effort. It should be pointed out that the factors of safety are somewhat sensitive to the distribution chosen; however, the other potential distributions give factors of safety that correspond in principle (large with respect to 1.0) if not in actual numerical value to those calculated above.

The Jablecki data were obtained for aircraft during the period when a simple static test (no thermal, dynamic, or fatigue problems were recognized) was generally sufficient to define the strength level. It might be questioned whether the Jablecki data are applicable to modern aircraft. Experience to date with modern aircraft structures has disclosed sufficient numbers of test and operational failures to conclude that this kind of data can be used as an indicator of the reliability of analytical designs. In addition, it should be remembered that analytical structural design is continually extending the boundaries of the technology. Again, experience has indicated that analysis is based on mathematical models which are imperfect and for this reason the Jablecki function yields approximations consistent with current analytical practice. Therefore, it is postulated that the Jablecki data represent an indicator of the capability for accurate analysis of advanced designs. On this basis, the Jablecki data are believed to be applicable to modern aircraft.

It has come to the author's attention that yet another study by Walker[8] on British aircraft has been made. Preliminary analysis of Walker's report by the author indicates that these data agree in general with those of Jablecki.

It is recommended that more detailed studies be performed to determine the reliability of analytical stress analysis procedures and methods. The structural reliability parameter values reported in this analysis indicate the no static test condition of the structure prior to static testing. In the course of developing a sound structural design criteria methodology and philosophy, a knowledge of structural reliability parameters of the kind presented in this paper can be invaluable.

It is apparent that to ignore the unreliability of stress analysis procedures can lead to serious trouble with components whose strength is not verified by test. The results presented in this paper demonstrate that, if reliance is placed only on analytical structural design, the required design factors become prohibitive. Although this fact has been assumed for many years throughout the aerospace industry, the Jablecki analysis, the Walker analysis, and the results of this paper indicate that structural testing is an absolute requirement for any complex system.

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